hep-ph/9604309 DESY-95-252; TUIMP-TH-96/75; MSUHEP-60405

March, 1996

Global Power Counting Analysis On Probing Electroweak Symmetry Breaking Mechanism At High Energy Colliders

Hong-Jian He [a], **Yu-Ping Kuang** [b], **C.–P. Yuan** [c]

[a] *Theory Division, DESY, D-22603 Hamburg, Germany*

[b] *CCAST (World Laboratory), P.O.Box 8730, Beijing 100080, China*
Institute of Modern Physics, Tsinghua University, Beijing 100084, China [†]

[c] *Department of Physics and Astronomy, Michigan State University*
East Lansing, Michigan 48824 , USA

Abstract

We develop a precise power counting rule (a generalization of Weinberg's counting method for the nonlinear sigma model) for the electroweak theories formulated by chiral Lagrangians. Then we estimate the contributions of *all* next-to-leading order (NLO) bosonic operators to the amplitudes of the relevant scattering processes which can be measured at high energy colliders, such as the LHC and future Linear Colliders. Based upon these results, we globally classify the sensitivities of testing all NLO bosonic operators for probing the electroweak symmetry breaking mechanism at high energy colliders.

PACS number(s): 11.30.Qc, 11.15.Ex, 12.15.Ji, 14.70.-e

(Published in Physics Letters **B382** (1996) 149-156)

[†]Mailing address.

1. Effective Lagrangian for Strongly Interacting EWSB Sector

The current low energy data are sensitive to the $SU(2)_L \times U(1)_Y$ gauge interactions of the Standard Model (SM), but still allow a wide mass-range (65.2 GeV$\sim O(1)$ TeV) for the SM Higgs boson [1] so that the electroweak symmetry breaking (EWSB) mechanism remains an open question. The light resonance(s) originating from the EWSB sector with mass(es) well below the TeV scale can exist possibly in the SM and necessarily in its supersymmetric extensions. In such cases, these particles should be detected [2, 3] at the high energy colliders such as the CERN Large Hadron Collider (LHC) and the future electron (and photon) Linear Colliders (LC) [4], even though the current direct experimental searches so far are all negative. If the EWSB is, however, driven by strong interactions with no new resonance well below the TeV scale, then it will be a greater challenge to future colliders to decisively probe the EWSB mechanism. This latter case is what we shall study in this work.

It is known that below the scale of any new heavy resonance the electroweak chiral Lagrangian (EWCL) provides the most economical method to describe the new physics effects, and is one of the most important applications of the general idea about effective field theories [5]. Following Ref. [6, 7], the EWCL can be written as

$$\mathcal{L}_{eff} = \sum_n \ell_n \frac{f_\pi{}^{r_n}}{\Lambda^{a_n}} \mathcal{O}_n(W_{\mu\nu}, B_{\mu\nu}, D_\mu U, U, f, \bar{f}) = \mathcal{L}_G + \mathcal{L}_S + \mathcal{L}_F \, , \tag{1}$$

where $D_\mu U = \partial_\mu U + ig\mathbf{W}_\mu U - ig' U \mathbf{B}_\mu$, $\mathbf{W}_\mu \equiv W_\mu^a \frac{\tau^a}{2}$, $\mathbf{B}_\mu \equiv B_\mu \frac{\tau^3}{2}$, $U = \exp[i\tau^a \pi^a / f_\pi]$, π^a is the Goldstone boson (GB) field and $f(\bar{f})$ is the fermion field. In (1), we have factorized out the dependence on f_π and Λ so that the dimensionless coefficient ℓ_n of the operators $\mathcal{O}_n$ are of $O(1)$ [8]. $f_\pi = 246\,\text{GeV}$ is the vacuum expectation value which characterizes the EWSB breaking scale. The effective cut-off scale Λ is the highest energy scale below which (1) is valid. In the case with no new light resonance in the EWSB sector, $\Lambda \approx 4\pi f_\pi$ [8]. $\mathcal{L}_F$ is the fermionic part of $\mathcal{L}_{eff}$.[a] The bosonic part of the EWCL is given by $\mathcal{L}_G + \mathcal{L}_S$ where $\mathcal{L}_G = -\frac{1}{2}\text{Tr}(\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$ and $\mathcal{L}_S$ contains operators describing the gauge-boson-GB interactions and the GB self-interactions:

$$\mathcal{L}_S \;=\; \mathcal{L}^{(2)} + \mathcal{L}^{(2)\prime} + \sum_{n=1}^{14} \mathcal{L}_n \; . \tag{2}$$

[a] Here we concentrate on probing new physics from all possible bosonic operators and do not include the next-to-leading order fermionic operators in $\mathcal{L}_F$.

$\mathcal{L}^{(2)}$ is the universal leading order bosonic operator, and equals to $\frac{f_\pi^2}{4}\mathrm{Tr}[(D_\mu U)^\dagger(D^\mu U)]$. All the other 15 next-to-leading-order (NLO) bosonic operators were explicitly given in Refs. [6, 7], among which twelve ($\mathcal{L}^{(2)\prime}$ and $\mathcal{L}_{1\sim 11}$) are CP-conserving and three ($\mathcal{L}_{12\sim 14}$) are CP-violating. Furthermore, the operators $\mathcal{L}_{6,7,10}$ violate custodial $SU(2)_C$ symmetry (even after g' is turned off) in contrast to the operators $\mathcal{L}_{4,5}$ in which the pure GB interactions are $SU(2)_C$-invariant.

The coefficients (ℓ_n's) of the 15 NLO operators depend on the details of the underlying dynamics and reflect the possible new physics. Among the 15 NLO coefficients, ℓ_1, ℓ_0 and ℓ_8 correspond to S, T and U parameters [6]. (S $= -\ell_1/\pi$, T $= \ell_0/(2\pi e^2)$ and U $= -\ell_8/\pi$.) They have been measured from the current low energy LEP/SLC data and will be further improved at LEPII and upgraded Tevatron. To distinguish different models of the EWSB, the rest of the ℓ_n's has to be measured by studying the scattering processes involving weak gauge bosons. What is usually done in the literature is to consider only a small subset of these operators at a time. For instance, in Ref. [2], a non-resonant model (called Delay-K model) was studied which includes $\mathcal{L}^{(2)}$ as well as the NLO operators $\mathcal{L}_4$ and $\mathcal{L}_5$. It was found that for the gold-plated mode (i.e. pure leptonic decay mode) of $W^\pm W^\pm$, a total number of about 10 signal events is expected at the LHC with a $100\,\mathrm{fb}^{-1}$ luminosity after imposing relevant kinematic cuts to suppress backgrounds. In the end of the analysis the ratio of signal to background is about 1. Another non-resonant model (called LET-CG model), which contains only the model-independent operator $\mathcal{L}^{(2)}$, was also studied in that paper. The difference between the predictions of these two models signals the effects from the NLO operators $\mathcal{L}_{4,5}$. With just a handful events, it requires higher integrated luminosities to probe these NLO operators and compare with the model-independent contributions from $\mathcal{L}^{(2)}$. Generally speaking, if one combines measurements from various VV-modes, it is possible (although not easy) to distinguish models of EWSB which effectively include different subsets of the 15 NLO operators and the model-independent operator $\mathcal{L}^{(2)}$.

The important question to ask is: " How and to what extent can one measure *all* the NLO coefficients ℓ_n at future colliders to *fully* explore the EWSB sector? " To answer this question, as the first step, one should **(i)**. find out, for each given NLO operator, whether it can be measured via leading and/or sub-leading amplitudes of relevant processes at each collider; **(ii)**. determine whether a given NLO operator can be sensitively (or marginally sensitively) probed through its contributions to the leading (or sub-leading) amplitudes of the relevant scattering process at each given collider; **(iii)**.

determine whether carrying out the above study for various high energy colliders can *complementarily* cover all the 15 NLO operators to probe the strongly interacting EWSB sector. For abbreviation, the above requirements **(i)**-**(iii)** will be referred hereafter as the " *Minimal Requirements* ".

To find the relevant scattering processes and determine their sensitivities to a given NLO operator, one has to first know the contributions of this operator to the scattering amplitudes under consideration. Although one can easily realize whether a single scattering process is relevant to probing a given NLO operator or not, it is non-trivial to classify all relevant processes to every NLO operator at different high energy colliders, and to further determine whether each given NLO operator can be sensitively/marginally sensitively probed by the corresponding scattering processes at these colliders. This would in principle require detailed calculations on the contributions of these operators to various scattering amplitudes. In this work, as a first-step global analysis, we shall only *estimate* the contributions of all these NLO operators to various scattering processes by using a power counting method constructed in Sec. 2. In Sec. 3, we examine the hierarchy structure for the sizes of the scattering amplitudes and define our theoretical criterion for classifying the sensitivities of relevant scattering processes to each NLO operator. These will answer our *Minimal Requirements*-**(i)** and -**(ii)**. Finally, given the above results, we globally and qualitatively classify, in Sec. 4, the sensitivities of the relevant scattering processes for probing all the NLO operators at relevant high energy colliders. This completes our answer to the *Minimal Requirement*-**(iii)**.

2. A Power Counting Rule for High Energy Scattering Amplitudes

To make a systematic analysis on the sensitivity of a scattering process for probing the new physics operators in (1), we have to first compute the scattering amplitudes contributed by those operators. For this purpose, we generalize Weinberg's power counting rule for the ungauged nonlinear sigma model (NLSM) [9] and develop a power counting rule for the EWCL to *separately* count the power dependences on the energy E and all the relevant mass scales. Weinberg's counting rule was to count the E-power dependence (D_E) for a given L-loop level S-matrix element T in the NLSM. To generalize it to the EWCL, we further include the gauge bosons, ghost bosons, fermions and possible v_μ-factors associated with external weak gauge boson ($V = W^\pm, Z^0$) lines, [cf. (6)]. After some algebra, we find that for the EWCL and in the energy region $\Lambda > E \gg M_W, m_t$,

$$D_E = 2L + 2 + \sum_n \mathcal{V}_n \left(d_n + \frac{1}{2} f_n - 2 \right) - e_v \quad , \tag{3}$$

where $\mathcal{V}_n$ is the number of type-n vertices in T, $d_n(f_n)$ is the number of derivatives (fermion-lines) at a vertex of type-n, and e_v is the number of possible external v^μ-factors [c.f. (6)]. For external fermions, we consider masses $m_f \le m_t \sim O(M_W) \ll E$, and the spinor wave functions are normalized as $\bar{u}(p,s)u(p,s') = 2m_f\delta_{ss'}$, etc.

To correctly estimate the magnitude of each given amplitude T, besides counting the power of E, it is also crucial to *separately* count the power dependences on the two typical mass scales of the EWCL: the vacuum expectation value f_π and the effective cut-off Λ of the effective theory.[b] The Λ-dependence comes from the NLO operator tree-level vertices, each of which contributes a factor $1/\Lambda^{a_n}$ [cf. (1)] so that the total factor is $1/\Lambda^{\sum_n a_n}$. The power factor Λ^{a_n} associated with each operator $\mathcal{O}_n$ can be counted by the naive dimensional analysis (NDA) [8]. In general, T can always be written as $f_\pi^{D_T}$ times some dimensionless function of E, Λ and f_π, where $D_T = 4 - e$ and e is the number of external bosonic and fermionic lines. Bearing in mind the intrinsic L-loop factor $(\frac{1}{16\pi^2})^L = (\frac{1}{4\pi})^{2L}$, we can then construct the following precise counting rule for T in the energy region $\Lambda > E \gg M_W, m_t$:

$$T = c_T f_\pi^{D_T} \left(\frac{f_\pi}{\Lambda}\right)^{N_\mathcal{O}} \left(\frac{E}{f_\pi}\right)^{D_{E0}} \left(\frac{E}{4\pi f_\pi}\right)^{D_{EL}} \left(\frac{M_W}{E}\right)^{e_v} H(\ln E/\mu) \;,$$

$$N_\mathcal{O} = \sum_n a_n \;, \qquad D_{E0} = 2 + \sum_n \mathcal{V}_n\left(d_n + \frac{1}{2}f_n - 2\right) \;, \qquad D_{EL} = 2L \;, \tag{4}$$

where the dimensionless coefficient c_T contains possible powers of gauge couplings (g, g') and Yukawa couplings (y_f) from the vertices of T, which can be directly counted. H is a function of $\ln(E/\mu)$ coming from loop corrections in the standard dimensional regularization [5] and is insensitive to E. Neglecting the insensitive factor $H(\ln E/\mu)$, we can extract the main features of scattering amplitudes by simply applying (4) to the corresponding Feynman diagrams.

Note that the counting for E-power dependence in (3) or (4) cannot be directly applied to the amplitudes with external longitudinal gauge boson (V_L) lines. Consider the tree-level $V_LV_L \to V_LV_L$ amplitude. Using (4) and adding the E-factors from the four longitudinal polarization vectors $\epsilon_L^\mu \sim k^\mu/M_{W,Z}$, we find that the leading amplitude is

[b] If the powers of f_π and Λ are not separately counted, $\Lambda/f_\pi \simeq 4\pi$ will be mistakenly counted as 1. This makes the estimated results off by orders of magnitude. If a power counting rule only counts the sum $D_E + D_\Lambda$ [10], it cannot be used to correctly estimate the order of magnitudes. E.g., the amplitudes $\frac{E^2}{f_\pi^2}$ and $\frac{E^2}{f_\pi^2}\frac{E^2}{\Lambda^2}$ have the *same* $D_E + D_\Lambda$ but are clearly of different orders in magnitude. (For $E = 1$ TeV, they differ by a factor ~ 10.)

proportional to E^4/f_π^4 which violates the low energy theorem result (i.e. E^2/f_π^2). This is because the naive power counting for V_L-amplitudes only gives the leading E-power of individual Feynman diagrams, it does not reflect the fact that gauge invariance causes the cancellations of the E^4-terms among individual diagrams. So, how can we count D_E in any amplitude with external V_L-lines? We find that this can be elegantly solved by using the Ward-Takahashi (WT) identity (cf. [11] for a precise derivation[c]):

$$T[V_L^{a_1},\cdots,V_L^{a_n};\Phi_\alpha] = C\cdot T[-i\pi^{a_1},\cdots,-i\pi^{a_n};\Phi_\alpha] + B \quad , \tag{5}$$

with
$$C \equiv C_{mod}^{a_1}\cdots C_{mod}^{a_n} \ , \quad v^a \equiv v^\mu V_\mu^a \ , \quad v^\mu \ \equiv \epsilon_L^\mu - k^\mu/M_a = O(M_a/E) \ ,$$

$$B \equiv \sum_{l=1}^{n}\{\ C_{mod}^{a_{l+1}}\cdots C_{mod}^{a_n}\, T[v^{a_1},\cdots,v^{a_l},-i\pi^{a_{l+1}},\cdots,-i\pi^{a_n};\Phi_\alpha] + \text{permutations}\ \}\ , \tag{6}$$

where π^a are GB fields and Φ_α denotes other possible physical in/out states. The constant modification factor $C_{mod}^a = 1 + O(\frac{g^2}{16\pi^2})$ in the EWCL and can be exactly simplified as 1 in certain convenient renormalization schemes [11]. Since the right-hand side (RHS) of (5) does not have E-power cancellations related to external legs, we can therefore apply our counting rule (4) to *indirectly count the D_E of the V_L-amplitude via counting the D_E of the RHS of (5).*

3. Estimating Scattering Amplitudes and Analyzing Their Sensitivities to Each Given Operator

The main advantage of using the above counting rule (4) is that we can correctly and quickly estimate the magnitude of any scattering amplitude in the energy region $M_W, m_t \ll E < \Lambda$. Using the above counting rule (4), we have performed a global analysis for all $V^aV^b \to V^cV^d$ and $q\bar{q} \to V^aV^b$ processes by estimating the contributions from both model-independent operator $\mathcal{L}_0(\equiv \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}^{(2)}\ )$ up to one-loop and the other 15 model-dependent NLO operators at tree-level [7]. We observe a power counting hierarchy in terms of E, f_π and Λ for these amplitudes:

$$\frac{E^2}{f_\pi^2} \gg \frac{E^2}{f_\pi^2}\frac{E^2}{\Lambda^2},\ g\frac{E}{f_\pi} \gg g\frac{E}{f_\pi}\frac{E^2}{\Lambda^2},\ g^2 \gg g^2\frac{E^2}{\Lambda^2},\ g^3\frac{f_\pi}{E} \gg g^3\frac{Ef_\pi}{\Lambda^2},\ g^4\frac{f_\pi^2}{E^2} \gg g^4\frac{f_\pi^2}{\Lambda^2}\ , \tag{7}$$

which, in the typical high energy region $E \in (750\,\text{GeV},\ 1.5\,\text{TeV})$, gives

$$\begin{gathered}(9.3,37) \gg (0.55,8.8),(2.0,4.0) \gg (0.12,0.93),(0.42,0.42) \gg \\ (0.025,0.099),(0.089,0.045) \gg (5.3,10.5)\times 10^{-3},(19.0,4.7)\times 10^{-3} \gg (1.1,1.1)\times 10^{-3}\end{gathered}\ ,$$

[c] This identity was used to derive the Equivalence Theorem (ET) to all orders in the perturbative expansion and to explore the profound physical content of the ET [11].

where E is taken to be the VV-pair invariant mass and $\Lambda \approx 4\pi f_\pi$. This power counting hierarchy is easy to understand. In (7), from left to right, the hierarchy is built up by increasing either the number of derivatives (i.e. power of E/Λ) or the number of external transverse gauge boson V_T's (i.e. the power of gauge couplings). This power counting hierarchy provides us a theoretical base to classify all the relevant scattering amplitudes in terms of the three essential parameters E, f_π and Λ plus possible gauge/Yukawa coupling constants. In the high energy region $M_W, m_t \ll E < \Lambda$ and to each order of chiral perturbation, for a given type of processes [which all contain the same number of external V-lines ($V = W^\pm$, or, Z) with other external lines exactly the same], the leading amplitude is given by the amplitude with all external V-lines being longitudinal, and the sub-leading amplitude is given by the amplitude with only one external V_T-line (and all other external V-lines being longitudinal). This is because the EWCL formalism is a momentum-expansion and the GBs (and thus V_L's) are derivatively coupled.

To answer the *Minimal Requirement*-**(i)**, we classify in Table 2 the most important leading and sub-leading amplitudes that can probe the NLO operators via various processes.[d] To answer the *Minimal Requirement*-**(ii)**, we shall establish a theoretical criterion for classifying the *sensitivity* of a given scattering process to each NLO operator.

Let us consider the scattering process $W^\pm W^\pm \to W^\pm W^\pm$ as a typical example to illustrate the idea. The leading and sub-leading amplitudes for this process are given by the one with four external W_L-lines ($T[4W_L]$) and the one with three external W_L-lines plus one W_T-line ($T[3W_L, W_T]$), respectively. In Table 1a we estimate the tree and one-loop level contributions from the model-independent operators in $\mathcal{L}_0 \equiv \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}^{(2)}$ to the leading amplitude $T[4W_L]$ and to the sub-leading amplitude $T[3W_L, W_T]$. In the same table, we also list the model-independent contributions to various B-terms [cf. (6)]. ($B^{(0)}$ and $B^{(1)}$ denote the B-term from V_L-amplitudes with 0 and 1 external V_T-line, respectively.) In Table 1b, we list the tree-level contributions from the model-dependent operators to these two amplitudes. For instance, the model-dependent leading contributions in $T[4W_L]$ come from the operators $\mathcal{L}_{4,5}$. (The contributions from $\mathcal{L}_{2,3,9}$ in $T[4W_L]$ are suppressed by a factor E^2/f_π^2 relative to that from $\mathcal{L}_{4,5}$.) Therefore, it is easier to measure $\mathcal{L}_{4,5}$ than $\mathcal{L}_{2,3,9}$ via the $W_L^\pm W_L^\pm \to W_L^\pm W_L^\pm$ process. From Table 1b, we also learn that the largest contributions in the sub-leading amplitude $T[3W_L, W_T]$ come from $\mathcal{L}_{3,4,5,9,11,12}$. To determine which operators can be sensitively probed via a given process, we introduce the following theoretical criterion on the sensitivity of the

[d] Other amplitudes below the sub-leading amplitude for each type of processes are given elsewhere [7].

process to probing a NLO operator. Consider the contributions of $\mathcal{L}_{4,5}$ to $T[4W_L]$ as an example. For this case, the WT identity (5) gives,

$$T[4W_L] = C \cdot T[4\pi] + B \quad , \tag{8}$$

where $C = 1 + O(\frac{g^2}{16\pi^2})$, $T[4\pi] = T_0[4\pi] + T_1[4\pi]$ and $B = B_0^{(0)} + B_1^{(0)}$, in which $T_1[4\pi]$ contains both the model-independent [$E^4/(16\pi^2 f_\pi^4)$] and model-dependent contributions [$\ell_{4,5}E^4/(f_\pi^2\Lambda^2)$], cf. Table 1a,b. Similarly, $B_1^{(0)}$ contains both the model-independent [$g^2E^2/(16\pi^2 f_\pi^2)$] and model-dependent [$\ell_{4,5}\, g^2E^2/\Lambda^2$] contributions. Note that the leading B-term $B_0^{(0)}$, which is of $O(g^2)$, only depends on the SM gauge couplings and is of the same order as the leading pure W_T-amplitude $T[4W_T]$ [11, 7]. Thus, B *is insensitive to the EWSB mechanism.* To *sensitively probe* the EWSB sector by measuring $\mathcal{L}_{4,5}$ via $T[4W_L]$ amplitude, we demand the pure GB-amplitude $T[4\pi]$ contributed from $\ell_{4,5}$ (as a direct reflection of the EWSB dynamics) to dominate over the corresponding model-independent leading B-term ($B_0^{(0)}$), i.e. reguiring $\ell_{4,5}E^4/(f_\pi^2\Lambda^2) \gg g^2$. This gives, for $\ell_{4,5} = O(1)$, $\frac{1}{4}\frac{E^2}{\Lambda^2} \gg \frac{M_W^2}{E^2}$, or $1 \gg (0.7\,\mathrm{TeV}/E)^4$.[e] Thus, *sensitively probing* $\mathcal{L}_{4,5}$ via the $4W_L^\pm$-process requires $E \geq 1\,\mathrm{TeV}$, which agrees with the conclusion from a detailed Monte Carlo study in Ref. [2].

It is straightforward to generalize the above discussion to any scattering process up to the NLO. In this paper, we generally classify the sensitivities of the processes as follows. For a scattering process involving the NLO coefficient ℓ_n, if $T_1 \gg B$, then this process is classified to be *sensitive* to the operator $\mathcal{L}_n$. If not, this process is classified to be either *marginally sensitive* (for $T_1 > B$ but $T_1 \not\gg B$) or *insensitive* (for $T_1 \leq B$) to the operator $\mathcal{L}_n$. In Tables 1 and 2, *both the GB-amplitude and the B-term are explicitly estimated by our counting rule (4).* If $T_1 \leq B$, this means that the sensitivity is poor so that the probe of T_1 is experimentally harder and requires a higher experimental precision of at least $O(B)$ to test T_1. The issue of whether to numerically include B in an explicit calculation of the V_L-amplitude is *irrelevant* to the above conclusion.

4. Classification of Sensitivities to Probing EWSB Sector at Future High Energy Colliders

[e] This condition was first correctly derived in the 1st paper of Ref. [11] for the EWCL and is different from that in Ref. [12] where the B-term was incorrectly estimated as $O(M_W/E)$ instead of $O(g^2)$. Also, f_π and Λ were not separately counted for T_0 and T_1 in Ref. [12] so that the factor $\frac{\Lambda^2}{f_\pi^2}$ ($\approx 16\pi^2 \geq O(10^2)$) was mistaken as 1. After private communications, the authors of Ref. [12] informed us that they agreed with our condition (see footnote-20 in the 1st paper of Ref. [11]).

This section is devoted to discuss our *Minimal Requirement*-**(iii)**. It is understood that the actual sensitivity of a collider to probe the NLO operators depends not only on the luminosities of the active partons (including weak-gauge bosons) inside hadrons or electrons (as discussed in Ref. [7]), but also on the detection efficiency of the signal events after applying background-suppressing kinematic cuts to observe the specific decay mode of the final state weak-bosons (as discussed in Refs. [2, 3]). However, all of these will only add fine structures to the sub-leading contributions listed in Table 2 but not affect our conclusions about the leading contributions as long as there are enough signal events produced. This fact was illustrated in Ref. [7] for probing the NLO operators via $W^{\pm}W^{\pm} \to W^{\pm}W^{\pm}$ at the LHC. We have further applied the same method to other scattering processes (including possible incoming photon/fermion fields) for various high energy colliders with the luminosities of the active partons included, the details of the study will be given elsewhere. In this paper, we shall not perform a detailed numerical study like Refs. [2, 3], but only give a first-step qualitative global power counting analysis which serves as a useful guideline for further elaborating numerical calculations.

After examining all the relevant $2 \to 2$ and $2 \to 3$ hard scattering processes, we summarize in Table 2 our global classification for the sensitivities of various future high energy colliders to probing the 15 model-dependent NLO bosonic operators. Here, the energy-E represents the typical energy scale of the hard scattering processes under consideration. The leading B-term for each high energy process is also listed and compared with the corresponding V_L-amplitude. If the polarizations of the initial/final state gauge bosons are not distinguished but simply summed up, the largest B in each process (including all possible polarization states) should be considered for comparison. [If the leading B_0 (with just one v_μ-factor, cf. eq. (6)) happens to be zero, then the largest next-to-leading term, either the part of B_0 term that contains 2 (or 3) v_μ-factors or the B_1 term, should be considered. Examples are the $ZZ \to ZZ$ and $f\bar{f} \to ZZZ$ processes.] By comparing T_1 with B in Table 2 and applying our criterion for classifying the sensitivities, we find that for the typical energy scale (E) of the relevant processes at each collider, the leading contributions (marked by $\surd$) can be sensitively probed, while the sub-leading contributions (marked by $\triangle$) can only be marginally sensitively probed.[f] (To save space, Table 2 does not list those processes to which the NLO operators *only* contribute sub-leading amplitudes. These processes are $WW \to W\gamma, Z\gamma +$ perm. and

[f] The exceptions are $f\bar{f}^{(\prime)} \to W^+W^-/(LT), W^{\pm}Z/(LT)$ for which $T_1 \leq B_0$. Thus the probe of them is insensitive. (L/T denotes the longitudinal/transverse polarizations of $W^{\pm}$, Z^0 bosons.)

$f\bar{f}^{(\prime)} \rightarrow W\gamma, WW\gamma, WZ\gamma$, which all have one external transverse γ-line and are at most marginally sensitive.)

From Table 2, some of our conclusions can be drawn as follows.

(1). At LC(0.5), which is a LC with $\sqrt{S} = 0.5\,\mathrm{TeV}$, $\ell_{2,3,9}$ can be sensitively probed via $e^-e^+ \rightarrow W_L^- W_L^+$.

(2). For pure $V_L V_L \rightarrow V_L V_L$ scattering amplitudes, the model-dependent operators $\mathcal{L}_{4,5}$ and $\mathcal{L}_{6,7}$ can be probed most sensitively. ℓ_{10} can only be sensitively probed via the scattering process $Z_L Z_L \rightarrow Z_L Z_L$ which is easier to detect at the LC(1.5) [a e^-e^+ or e^-e^- collider with $\sqrt{S} = 1.5\,\mathrm{TeV}$] than at the LHC(14) [a pp collider with $\sqrt{S} = 14\,\mathrm{TeV}$].

(3). The contributions from $\mathcal{L}^{(2)\prime}$ and $\mathcal{L}_{2,3,9}$ to the pure $4V_L$-scattering processes lose the E-power dependence by a factor of 2 (see, e.g., Table 1b). Hence, the pure $4V_L$-channel is less sensitive to these operators. [Note that $\mathcal{L}_{2,3,9}$ can be sensitively probed via $f\bar{f} \rightarrow W_L^- W_L^+$ process at LC(0.5) and LHC(14).] The pure $4V_L$-channel cannot probe $\mathcal{L}_{1,8,11\sim 14}$ which can only be probed via processes with V_T('s). Among $\mathcal{L}_{1,8,11\sim 14}$, the contributions from $\mathcal{L}_{11,12}$ to processes with V_T('s) are most important, although their contributions are relatively suppressed by a factor gf_π/E as compared to the leading contributions from $\mathcal{L}_{4,5}$ to pure $4V_L$-scatterings. $\mathcal{L}_{1,8,13,14}$ are generally suppressed by higher powers of gf_π/E and are thus the least sensitive. The above conclusions hold for both LHC(14) and LC(1.5).

(4). At LHC(14), $\ell_{11,12}$ can be sensitively probed via $q\bar{q}' \rightarrow W^\pm Z$ whose final state is not electrically neutral. Thus, this final state is not accessible at LC. Hence, LC(0.5) will not be sensitive to these operators. To sensitively probe $\ell_{11,12}$ at LC(1.5), one has to measure $e^-e^+ \rightarrow W_L^- W_L^+ Z_L$.

(5). To sensitively probe $\ell_{13,14}$, a high energy $e^-\gamma$ linear collider is needed for studying the processes $e^-\gamma \rightarrow \nu_e W_L^- Z_L,\ e^- W_L^- W_L^+$, in which the backgrounds [13] are much less severe than processes like $\gamma\gamma \rightarrow W_L^+ W_L^-$ at a $\gamma\gamma$ collider [14, 4].[g]

From the above global analysis, we speculate[h] that before having a large number of signal events at the LHC (i.e. with large integrated luminosity), the LHC alone will not be able to sensitively measure all these operators, the linear collider is needed to *complementarily* cover the rest of the NLO operators. In fact, the different phases of

[g] The amplitude of $\gamma\gamma \rightarrow W_L^+ W_L^-$ has the order of $e^2\frac{E^2}{\Lambda^2}$, to which the $\mathcal{L}_{13,14}$ (and also $\mathcal{L}_{1,2,3,8,9}$) can contribute. Thus, this process would be useful for probing $\ell_{13,14}$ at a $\gamma\gamma$ collider if the backgrounds somehow could be efficiently suppressed.

[h] To further reach a detailed quantitative conclusion, an elaborate and precise numerical study on all signal/background rates is necessary.

500 GeV and 1.5 TeV energies at the LC are necessary because they will be sensitive to different NLO operators in the EWCL. An electron-photon (or a photon-photon) collider is also very useful for measuring all the NLO operators which distinguish different models of the EWSB in the strongly interacting scenario.

Acknowledgments We thank Mike Chanowitz, Tao Han, Francisco Larios, and Peter Zerwas for reading the manuscript and for useful suggestions. H.J.H. is supported by the AvH of Germany and the U.S. DOE; Y.P.K. by the NSF of China and the Tsinghua FRF; and C.P.Y. by the U.S. NSF.

References

1. *Higgs Physics At LEP2* , M. Carena and P.M. Zerwas (Conv.), hep-ph/9602250, Report on Physics at LEP2, Vol. 1, Ed. G. Altarelli et al, CERN, 1995.
2. J. Bagger, V. Barger, K. Cheung, J. Gunion, T. Han, G.A. Ladinsky, R. Rosenfeld, C.–P. Yuan, Phys. Rev. **D49** (1994) 1246; **D52** (1995) 3878.
3. V. Barger, J.F. Beacom, K. Cheung, T. Han, Phys. Rev. **D50** (1993) 6704.
4. E.g., *Physics and Experiments with Linear* e^+e^- *Colliders*, Waikoloa, USA, 1993, Ed. F.A. Harris et al; and M.S. Chanowitz, *Physics at High Energy* γ-γ *Colliders,* Nucl. Instru. & Methods **A355** (1995) 42 and hep-ph/9407231.
5. For a nice review, H. Georgi, Ann. Rev. Nucl. & Part. Sci. **43** (1994) 209.
6. T. Appelquist et al, Phys. Rev. **D48** (1993) 3235, and references therein.
7. H.-J. He, Y.-P. Kuang, C.-P. Yuan, DESY-96-148 (to appear in *Phys.Rev.D*) and hep-ph/9508295, and references therein.
8. H. Georgi, *Weak Interaction and Modern Particle Theory,* 1984;
   A. Manohar and H. Georgi, Nucl. Phys. **B234** (1984) 189.
9. S. Weinberg, Physica **96A** (1979) 327.
10. *E.g.*, H. Veltman, Phys. Rev. **D41** (1990) 2294; and C. Gross-Knetter, BI-TP/25.
11. H.-J. He, Y.-P. Kuang, C.-P. Yuan, Phys. Rev. **D51** (1995) 6463; H.-J. He, Y.-P. Kuang, X. Li, Phys. Rev. Lett. **69** (1992) 2619; Phys. Rev. **D49** (1994) 4842; Phys. Lett. **B329** (1994) 278; H.-J. He, W.B. Kilgore, DESY-96-079 and hep-ph/9609326 (to appear in *Phys.Rev.D*); and references therein.
12. A. Dobado, J.R. Pelaez, Phys. Lett. **B329** (1994) 469 (Errata: B335,554).
13. K. Cheung, S. Dawson, T. Han, G. Valencia, Phys. Rev. **D51** (1995) 5.
14. E.g., M. Golden, T. Han, G. Valencia, hep-ph/9511206, October, 1995.

Table 1. Estimates of leading/sub-leading amplitudes and the corresponding B-terms for $W^\pm W^\pm \to W^\pm W^\pm$ scatterings.

Table 1a. Model-independent contributions from $\mathcal{L}_G+\mathcal{L}_F+\mathcal{L}^{(2)}$ ($\Lambda_0 \equiv 4\pi f_\pi$).

$\mathcal{L}_G+\mathcal{L}_F+\mathcal{L}^{(2)}$	$T_\ell[4\pi]$	$T_\ell[3\pi, W_T]$	$B_\ell^{(0)}$	$B_\ell^{(1)}$
Tree-Level ($\ell=0$)	$\frac{E^2}{f_\pi^2}$	$g\frac{E}{f_\pi}$	g^2	$g^2\frac{M_W}{E}$
One-Loop ($\ell=1$)	$\frac{E^2}{f_\pi^2}\frac{E^2}{\Lambda_0^2}$	$g\frac{E}{f_\pi}\frac{E^2}{\Lambda_0^2}$	$g^2\frac{E^2}{\Lambda_0^2}$	$g^3\frac{Ef_\pi}{\Lambda_0^2}$

Table 1b. Model-dependent contributions from each NLO operator.
(Note: Up to the order $1/\Lambda^2$, $\mathcal{L}_{6,7,10}$ do not contribute to this process.)

Operators	$\mathcal{L}^{(2)\prime}$	$\mathcal{L}_{1,13}$	$\mathcal{L}_2$	$\mathcal{L}_3$	$\mathcal{L}_{4,5}$	$\mathcal{L}_{8,14}$	$\mathcal{L}_9$	$\mathcal{L}_{11,12}$
$T_1[4\pi]$	$\ell_0\ \frac{E^2}{\Lambda^2}$	/	$\ell_2\ e^2\frac{E^2}{\Lambda^2}$	$\ell_3\ g^2\frac{E^2}{\Lambda^2}$	$\ell_{4,5}\ \frac{E^2}{f_\pi^2}\frac{E^2}{\Lambda^2}$	/	$\ell_9\ g^2\frac{E^2}{\Lambda^2}$	/
$T_1[3\pi, W_T]$	$\ell_0\ g\frac{f_\pi E}{\Lambda^2}$	$\ell_{1,13}\ e^2 g\frac{f_\pi E}{\Lambda^2}$	$\ell_2\ e^2 g\frac{f_\pi E}{\Lambda^2}$	$\ell_3\ g\frac{E}{f_\pi}\frac{E^2}{\Lambda^2}$	$\ell_{4,5}\ g\frac{E}{f_\pi}\frac{E^2}{\Lambda^2}$	$\ell_{8,14}\ g^3\frac{f_\pi E}{\Lambda^2}$	$\ell_9\ g\frac{E}{f_\pi}\frac{E^2}{\Lambda^2}$	$\ell_{11,12}\ g\frac{E}{f_\pi}\frac{E^2}{\Lambda^2}$
$B_1^{(0)}$	$\ell_0\ g^2\frac{f_\pi^2}{\Lambda^2}$	$\ell_{1,13}\ e^2 g^2\frac{f_\pi^2}{\Lambda^2}$	$\ell_2\ e^2 g^2\frac{f_\pi^2}{\Lambda^2}$	$\ell_3\ g^2\frac{E^2}{\Lambda^2}$	$\ell_{4,5}\ g^2\frac{E^2}{\Lambda^2}$	$\ell_{8,14}\ g^4\frac{f_\pi^2}{\Lambda^2}$	$\ell_9\ g^2\frac{E^2}{\Lambda^2}$	$\ell_{11,12}\ g^2\frac{E^2}{\Lambda^2}$
$B_1^{(1)}$	$\ell_0\ g^3\frac{f_\pi^3}{\Lambda^2 E}$	$\ell_{1,13}\ e^4 g\frac{f_\pi^3}{\Lambda^2 E}$	$\ell_2\ e^2 g\frac{f_\pi E}{\Lambda^2}$	$\ell_3\ g^3\frac{f_\pi E}{\Lambda^2}$	$\ell_{4,5}\ g^3\frac{f_\pi E}{\Lambda^2}$	$\ell_{8,14}\ g^3\frac{f_\pi E}{\Lambda^2}$	$\ell_9\ g^3\frac{f_\pi E}{\Lambda^2}$	$\ell_{11,12}\ g^3\frac{f_\pi E}{\Lambda^2}$

Table 2. Probing the EWSB Sector at High Energy Colliders: A Global Classification for the NLO Bosonic Operators

(Notations: $\surd$ = Leading contributions, $\triangle$ = Sub-leading contributions, and $\perp$ = Low-energy contributions.)
(Notes: †Here, $\mathcal{L}_{13}$ or $\mathcal{L}_{14}$ does not contribute at $O(1/\Lambda^2)$. ‡At LHC(14), $W^+W^+ \to W^+W^+$ should also be included.)

Operators	$\mathcal{L}^{(2)\prime}$	$\mathcal{L}_{1,13}$	$\mathcal{L}_2$	$\mathcal{L}_3$	$\mathcal{L}_{4,5}$	$\mathcal{L}_{6,7}$	$\mathcal{L}_{8,14}$	$\mathcal{L}_9$	$\mathcal{L}_{10}$	$\mathcal{L}_{11,12}$	$T_1 \;\Vert\; B$	Processes
LEP-I (S,T,U)	$\perp$	$\perp^\dagger$					$\perp^\dagger$				$g^4\frac{f_\pi^2}{\Lambda^2}$	$e^-e^+ \to Z \to f\bar{f}$
LEP-II	$\perp$	$\perp$	$\perp$	$\perp$			$\perp$	$\perp$		$\perp$	$g^4\frac{f_\pi^2}{\Lambda^2}$	$e^-e^+ \to W^-W^+$
LC(0.5)/LHC(14)			$\surd$	$\surd$				$\surd$			$g^2\frac{E^2}{\Lambda^2} \Vert g^2\frac{M_W^2}{E^2}$	$f\bar{f} \to W^-W^+/(LL)$
		$\triangle$	$\triangle$	$\triangle$			$\triangle$	$\triangle$		$\triangle$	$g^3\frac{Ef_\pi}{\Lambda^2} \Vert g^2\frac{M_W}{E}$	$f\bar{f} \to W^-W^+/(LT)$
LC(1.5)/LHC(14)				$\surd$	$\surd$	$\surd$		$\surd$		$\surd$	$g^2\frac{1}{f_\pi}\frac{E^2}{\Lambda^2} \Vert g^3\frac{M_W}{E^2}$	$f\bar{f} \to W^-W^+Z/(LLL)$
		$\triangle$	$\triangle$	$\triangle$	$\triangle$	$\triangle$	$\triangle$	$\triangle$		$\triangle$	$g^3\frac{E}{\Lambda^2} \Vert g^3\frac{M_W^2}{E^3}$	$f\bar{f} \to W^-W^+Z/(LLT)$
				$\surd$	$\surd$	$\surd$			$\surd$		$g^2\frac{1}{f_\pi}\frac{E^2}{\Lambda^2} \Vert g^3\frac{M_W}{\Lambda^2}$	$f\bar{f} \to ZZZ/(LLL)$
					$\triangle$	$\triangle$			$\triangle$		$g^3\frac{E}{\Lambda^2} \Vert g^3\frac{f_\pi}{\Lambda^2}\frac{M_W}{E}$	$f\bar{f} \to ZZZ/(LLT)$
					$\surd$						$\frac{E^2}{f_\pi^2}\frac{E^2}{\Lambda^2} \Vert g^2$	$W^-W^\pm \to W^-W^\pm/(LLLL)$ ‡
				$\triangle$	$\triangle$			$\triangle$		$\triangle$	$g\frac{E}{f_\pi}\frac{E^2}{\Lambda^2} \Vert g^2\frac{M_W}{E}$	$W^-W^\pm \to W^-W^\pm/(LLLT)$ ‡
					$\surd$	$\surd$					$\frac{E^2}{f_\pi^2}\frac{E^2}{\Lambda^2} \Vert g^2$	$W^-W^+ \to ZZ$ & perm.$/(LLLL)$
			$\triangle$	$\triangle$	$\triangle$	$\triangle$		$\triangle$		$\triangle$	$g\frac{E}{f_\pi}\frac{E^2}{\Lambda^2} \Vert g^2\frac{M_W}{E}$	$W^-W^+ \to ZZ$ & perm.$/(LLLT)$
					$\surd$	$\surd$			$\surd$		$\frac{E^2}{f_\pi^2}\frac{E^2}{\Lambda^2} \Vert g^2\frac{E^2}{\Lambda^2}$	$ZZ \to ZZ/(LLLL)$
				$\triangle$	$\triangle$	$\triangle$			$\triangle$		$g\frac{E}{f_\pi}\frac{E^2}{\Lambda^2} \Vert g^2\frac{M_W E}{\Lambda^2}$	$ZZ \to ZZ/(LLLT)$
LHC(14)				$\surd$						$\surd$	$g^2\frac{E^2}{\Lambda^2} \Vert g^2\frac{M_W^2}{E^2}$	$q\bar{q}' \to W^\pm Z/(LL)$
		$\triangle$	$\triangle$	$\triangle$			$\triangle$	$\triangle$		$\triangle$	$g^3\frac{Ef_\pi}{\Lambda^2} \Vert g^2\frac{M_W}{E}$	$q\bar{q}' \to W^\pm Z/(LT)$
				$\surd$	$\surd$			$\surd$		$\surd$	$g^2\frac{1}{f_\pi}\frac{E^2}{\Lambda^2} \Vert g^3\frac{M_W}{E^2}$	$q\bar{q}' \to W^-W^+W^\pm/(LLL)$
			$\triangle$	$\triangle$	$\triangle$		$\triangle$	$\triangle$		$\triangle$	$g^3\frac{E}{\Lambda^2} \Vert g^3\frac{M_W^2}{E^3}$	$q\bar{q}' \to W^-W^+W^\pm/(LLT)$
				$\surd$	$\surd$	$\surd$				$\surd$	$g^2\frac{1}{f_\pi}\frac{E^2}{\Lambda^2} \Vert g^3\frac{M_W}{E^2}$	$q\bar{q}' \to W^\pm ZZ/(LLL)$
		$\triangle$	$\triangle$	$\triangle$	$\triangle$	$\triangle$	$\triangle$	$\triangle$		$\triangle$	$g^3\frac{E}{\Lambda^2} \Vert g^3\frac{M_W^2}{E^3}$	$q\bar{q}' \to W^\pm ZZ/(LLT)$
LC($e^-\gamma$)		$\surd$	$\surd$	$\surd$			$\surd$	$\surd$		$\surd$	$eg^2\frac{E}{\Lambda^2} \Vert eg^2\frac{M_W^2}{E^3}$	$e^-\gamma \to \nu_e W^-Z/(LL)$
		$\surd$	$\surd$	$\surd$			$\surd$	$\surd$		$\surd$	$eg^2\frac{E}{\Lambda^2} \Vert eg^2\frac{M_W^2}{E^3}$	$e^-\gamma \to e^-W^-W^+/(LL)$